Advance AI  
Ch15: Solution

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**Q 15.2** In this exercise, we examine what happens to the probabilities in the umbrella world in the limit of long time sequences.

**a**. Suppose we observe an unending sequence of days on which the umbrella appears. Show that, as the days go by, the probability of rain on the current day increases mono- tonically toward a fixed point. Calculate this fixed point.

**b**. Now consider *forecasting* further and further into the future, given just the first two umbrella observations. First, compute the probability P(r2+k|u1,u2) for k=1...20 and plot the results. You should see that the probability converges towards a fixed point. Prove that the exact value of this fixed point is 0.5.

Solution:-  
a) For all t, we have the filtering formula  
P(Rt|u1:t) = αP(ut|Rt) .

At the fixed point, we additionally expect that P(Rt|u1:t) = P(Rt−1|u1:t−1). Let the

fixed-point probabilities be ⟨ρ, 1 − ρ⟩. This provides us with a system of equations:

⟨ρ, 1 − ρ⟩ = α⟨0.9, 0.2⟩⟨0.7, 0.3⟩ρ + ⟨0.3, 0.7⟩(1 − ρ)

= α⟨0.9, 0.2⟩(⟨0.4ρ, −0.4ρ⟩ + ⟨0.3, 0.7⟩)

= ⟨0.9, 0.2⟩\*(⟨0.4ρ, −0.4ρ⟩ + ⟨0.3, 0.7⟩)

Solving this system, we find that ρ ≈ 0.8933.

b) The probability converges to ⟨0.5, 0.5⟩ .This convergence makes sense if we consider a fixed-point equation for P(R2+k|U1, U2):

P(R2+k|U1,U2) = ⟨0.7,0.3⟩P(r2+k−1|U1,U2)+⟨0.3,0.7⟩P(¬r2+k−1|U1,U2) P(r2+k|U1,U2) = 0.7P(r2+k−1|U1,U2)+0.3(1−P(r2+k−1|U1,U2))

= 0.4P(r2+k−1|U1,U2)+0.3

That is, P(r2+k|U1,U2) = 0.5.  
Notice that the fixed point does not depend on the initial evidence.

**15.13** A professor wants to know if students are getting enough sleep. Each day, the pro- fessor observes whether the students sleep in class, and whether they have red eyes. The professor has the following domain theory:

* The prior probability of getting enough sleep, with no observations, is 0.7.
* The probability of getting enough sleep on night t is 0.8 given that the student got

enough sleep the previous night, and 0.3 if not.

* The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
* The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

Formulate this information as a dynamic Bayesian network that the professor could use to filter or predict from a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.

Bottom of Form

Dynamic Bayesian Network:

The dynamic Bayesian network for this problem would have two time steps: t-1 and t. The nodes in the network would be as follows:

• S(t-1): Sleep status on night t-1 (can be "enough" or "not enough")

• S(t): Sleep status on night t (can be "enough" or "not enough")

• R(t): Red eyes on day t (can be "yes" or "no")

• C(t): Sleeping in class on day t (can be "yes" or "no")

The arrows in the network represent causal relationships between the nodes. The network would look like this:  
 S(t-1) S(t)

\ /

V

R(t)

|

V

C(t)

The conditional probability tables (CPTs) for the nodes would be as follows:

• P(S(t-1)): This would be given in the problem as a prior probability. The CPT would be:  
 | S(t-1) |

| enough | not enough |

|--------|------------|

| 0.7 | 0.3 |

P(S(t) | S(t-1)): This is the transition probability for the sleep status from night t-1 to night t. The CPT would be:

| S(t-1) | S(t) |

|--------|----------|

| enough | 0.8 0.2 |

|--------|----------|

| not enough | 0.3 0.7 |  
  
• P(R(t) | S(t)): This is the probability of having red eyes on day t given the sleep status on night t. The CPT would be:

| S(t) | R(t) |

|------|----------|

| enough | 0.2 0.8 |

|------|----------|

| not enough | 0.7 0.3 |

• P(C(t) | S(t)): This is the probability of sleeping in class on day t given the sleep status on night t. The CPT would be:

| S(t) | C(t) |

|------|----------|

| enough | 0.1 0.9 |

|------|----------|

| not enough | 0.3 0.7 |

Hidden Markov Model:

To reformulate the dynamic Bayesian network as a hidden Markov model with a single observation variable, we can collapse the R(t) and C(t) nodes into a single "observation" node O(t) with two possible values: "awake" (if the student did not sleep in class and does not have red eyes) or "tired" (if the student either slept in class or has red eyes). The HMM would look like this:

S(t-1) S(t)

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V

O(t)